Relativistic Electron-Beam-Generated Coherent Submillimeter Wavelength Cerenkov Radiation

J. E. WALSH, T. C. MARSHALL, M. R. MROSS, AND S. P. SCHLESINGER

***Abstract*—The possibility of using intense relativistic electron-beam-driven dielectric-loaded waveguides as submillimeter wavelength radiation sources is discussed.**

I. Introduction

We have in a recent series of experiments [1] obtained 1 MW of radiation at a wavelength of 4 mm from an electron-beam-driven dielectric-loaded waveguide. The relative simplicity, and certain other operating characteristics of the device, suggest that it may also warrant examination as a submillimeter wavelength source.

The idea of using the Cerenkov effect to make short wavelength radiation sources is not new [2]–[5]. The earliest work [2], [3], however, emphasized single particle emission, while later work [4], [5] aimed more specifically at microwaves did not make use of beams with an intensity sufficient to drive the instability which is the central feature of the current device.

In this paper we will divide the discussion into three parts. First we treat the case of the dielectric filled guide, second we examine the partially filled guide, and finally we briefly discuss some expected spectral and nonlinear properties of these devices.

II. The Filled Guide

The simplicity of the filled guide case will enable us to display the main features of the interaction. We restrict the analysis to azimuthally symmetric TM modes, and assume the presence of an axial (z) magnetic field strong enough to render the electron motion one dimensional. Seeking the usual traveling-wave solutions then enables us to reduce the problem to solving

$$\left[\frac{1}{r}\frac{d}{dr}r\frac{d}{dr} + \left(\frac{\omega^2\varepsilon}{c^2} - k^2\right)\varepsilon_{\|}\right]A(r) = 0. \tag{1}$$

The function $A(r)$ describes the radial dependence of the axial component of the electric field

$$E_z(r,z,t) = A(r)e^{i(kz-\omega t)} \tag{2a}$$

ε is the dielectric constant of the filling material, and

$$\varepsilon_{\|} = 1 - \frac{\omega_b^2}{(\omega - ck\beta)^2} \tag{2b}$$

is the beam-related dielectric function. Quantities not previously identified are $c\beta$, the beam velocity, and the "longitudinal plasma frequency"

$$\omega_b^2 = 4\pi ne^2/m\varepsilon\gamma^3. \tag{3}$$

In (3), n is the beam density in the laboratory frame, and $\delta^{-1} = \sqrt{1-\beta^2}$.

Since the axial field vanishes at the guide walls, $A(r)$ will be a zero-order Bessel function. Introducing a radial wave number p and the guide radius b where

$$\frac{1}{r}\frac{d}{dr}r\frac{d}{dr}J_0(pr) = -p^2J_0(pr) \tag{4a}$$

and

$$J_0(pb) = 0 \tag{4b}$$

enables us to extract a dispersion relation from (1). This is

$$\omega^2 - \omega_{0k}^2 - \frac{\omega_b^2(\omega^2 - c^2k^2/\varepsilon)}{(\omega - ck\beta)^2} = 0 \tag{5a}$$

where

$$\omega_{0k}^2 = c^2(p^2 + k^2)/\varepsilon \tag{5b}$$

is the dispersion relation of the guide in the absence of the beam. When $c\beta > 1/\sqrt{\varepsilon}$, the beam and the initially unperturbed guide mode ω_{0k} will be coupled [Fig. 1(a)]. At synchronism

$$ck\beta = \omega_{0k} \tag{6a}$$

which, taken together with (5b) also defines an axial wave number

$$k = p/\sqrt{\beta^2\varepsilon - 1}. \tag{6b}$$

This last relation is sketched in Fig. 1(b). We should note at this point that operating with $\beta^2\varepsilon$ not to much greater than unity will allow short wavelength operation with relatively large transverse guide dimension.

The roots of (5a) with the largest imaginary part occur near synchronism; accordingly, we let

$$\omega = ck\beta + \delta. \tag{7}$$

Before solving (5a), it will be convenient to rewrite the beam plasma frequency in terms of the dimensionless beam strength parameter ν (the number of electrons per unit length times the classical electron radius) and the beam radius b

$$\omega_b^2 = 4\nu c^2/\varepsilon\gamma^3b^2 \tag{8a}$$

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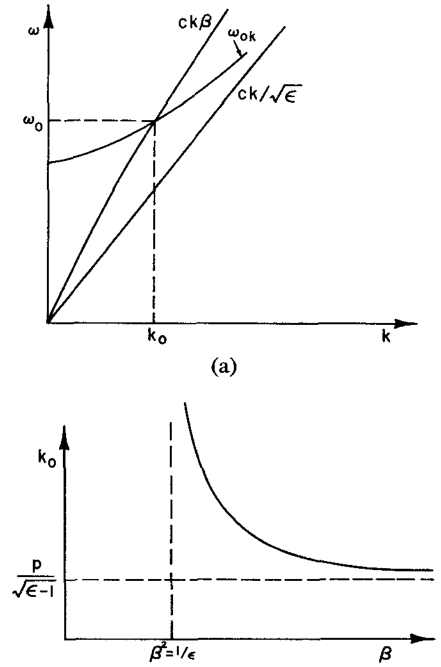

Fig. 1. (a) Sketches of the guide dispersion in the absence of a beam ω_{0k}, the beam frequency $ck\beta$, the asymptotic frequency $ck/\sqrt{\varepsilon}$, and the synchronous operating point k_0, ω_0. (b) Sketch of the synchronous wave number k_0 versus the relative beam velocity $\beta = v/c$.

and to also introduce the guide radius into p by letting

$$p = x_{0l}/b \tag{8b}$$

where x_{0l} is the lth root of J_0.

The dispersion relation, at synchronism and for weak-to-moderate beam strengths, $\nu/\gamma < 1$, then reduces to

$$\delta^3 = 2\nu x_{0l}(1 - 1/\beta^2\varepsilon)^{1/2}c/\varepsilon^{3/2}\gamma^3 b. \tag{9}$$

This equation has one real and two complex conjugate roots. The growth rate given by the imaginary part of δ is

$$\delta_I = \sqrt{3}(2\nu x_{0l}\sqrt{1 - 1/\beta^2\varepsilon})^{1/3}c/2\sqrt{\varepsilon}\,\gamma b. \tag{10}$$

An appreciation for the relative size of the growth rate can be obtained by comparing δ_I to the real part of the frequency ($\omega_R \simeq ck\beta$)

$$\frac{\delta_I}{ck\beta} = \frac{\sqrt{3}}{2}\left(\frac{2\nu}{x_{0l}^2}\right)^{1/3}\frac{(1 - 1/\beta^2\varepsilon)^{2/3}}{\gamma}. \tag{11}$$

Three things can be concluded of this point. First, as mentioned earlier, short wavelength operation with moderately large radius can be obtained by letting $\beta^2\varepsilon$ approach unity. This can be achieved while maintaining relatively large gain since the latter drops off only as $(1 - 1/\beta^2\varepsilon)^{1/6}$. Second, we should note that moderate beam strength will give reasonably large fractional gain, and hence the instability is strong. Third and finally, we point out that for beam energy relatively near the Cerenkov cutoff, a rapid tuning of center frequency with beam energy should result.

III. The Partially Filled Guide

We now assume a guide of radius b fitted with a dielectric sleeve of radial thickness $d = b - a$. The beam propagates in (the evacuated) central region. With the aid of standard procedures, we arrive at a dispersion relation of the system:

$$\frac{p}{\varepsilon_{\parallel}}\frac{J_0(pa)}{J_1(pa)} = \frac{q}{\varepsilon}\frac{J_0(qa)N_0(qb) - N_0(qa)J_0(qb)}{J_1(qa)N_0(qb) - N_1(qa)J_0(qb)} \tag{12a}$$

where

$$p^2 = \left(\frac{\omega^2}{c^2} - k^2\right)\varepsilon_{\parallel} \tag{12b}$$

and

$$q^2 = \left(\frac{\omega^2\varepsilon}{c^2} - k^2\right) \tag{12c}$$

define the radial mode numbers in the central region and the dielectric sleeve. The functions J and N are Bessel functions and modified Bessel functions, respectively.

The other symbols have the previous meanings with the exception that in ε the beam plasma frequency is now

$$\omega_b^2 = 4\pi ne^2/\gamma^3 m \tag{13a}$$

$$= 4\nu c^2/\gamma^3 a^2. \tag{13b}$$

In general, (12a) must now be solved numerically. However, when the dielectric shell is relatively thin and ε not too far from unity, the right-hand side of (12a) may be approximated by $\tan qd$ and the left-hand side may be expanded around a zero of $J_0(pa)$. Doing this and collecting terms, we arrive at the approximate dispersion relation

$$\omega^2 - \omega_k^2 - \frac{\omega_b^2(\omega^2 - c^2k^2\eta^2)}{(\omega - ck\beta)^2} = 0 \tag{14}$$

which is virtually identical to the one for the filled guide. The definitions of terms appearing in (14) are, however, slightly different. The dispersion in the absence of the beam is

$$\omega_k^2 = \omega_{c0}^2 + c^2k^2\eta^2 \tag{15a}$$

where

$$\eta = 1 + \frac{d}{a}\left(\frac{1}{\varepsilon} - 1\right) \tag{15b}$$

is a slowing factor which replaces $1/\varepsilon$ and

$$\omega_{c0}^2 = \left(\frac{2.4c}{a}\right)^2\Big/\left(1 + \frac{2d}{a}\right) \tag{15c}$$

is the cutoff frequency. The latter is correct only to lowest order in d/a.

More precise estimates could be made but for the purpose of indicating trends the above will suffice. Inspection of (14) shows that the roots occur in the same way that they do in (4a), and hence with the aid of the same definition of δ we can state that

$$\delta_I = \sqrt{3}(2.4\nu a\sqrt{1 - \eta^2/\beta^2/b})^{1/3}c/2a\gamma. \tag{16}$$

Synchronism occurs for a wave number,

$$k = 2.4/a\sqrt{\beta^2 - \eta^2} \tag{17}$$

and the ratio of imaginary-to-real frequencies is

$$\frac{\delta_I}{ck\beta} = \frac{\sqrt{3}}{2}\left(\frac{va}{5.76b}\right)^{1/3}\frac{(1-\eta^2/\beta^2)^{2/3}}{\gamma}. \tag{18}$$

It can be seen from (16)–(18) that the conclusions reached at the end of Section II are also true in the case of a partially filled guide. Furthermore, in addition to the obvious simplicity of having the beam propagate in a channel, the flexibility afforded by having η available as a design choice is convenient.

IV. Discussion

In addition to the conclusions mentioned at the end of Sections II and III, a few other observations can be made. First, the present system is similar in many respects to beam plasma systems and hence we would expect it to show similar spectral properties [7]. Narrow-band ($\Delta f/f < 0.05$) operation has already been observed at the longer wavelength, and it should as the gain is increased show first multiple mode and then wide-band radiation output. Judicious choice of axial reflections could, as is the case in a beam plasma system, also lead to mode-locked operation.

The present analysis assumes a monoenergetic electron beam. In the case of the relativistic beam, this requirement is less stringent than it is in the nonrelativistic limit. This follows because the velocity spread $\Delta\beta$ is fractionally smaller than the energy spread ($\Delta\beta = \Delta\varepsilon/\beta\gamma^3$). Furthermore, even in the case of a very warm beam the system is unstable and hence it will oscillate. The analysis in the warm limit is conceptually similar but more involved than the one presented above.

Finally, we should mention nonlinear effects and saturation. In the cold beam limit, saturation should occur by beam trapping and, based on experience with traveling-wave tubes and beam plasma systems, we might expect conversion efficiencies in the 10–30-percent range. The efficiency obtained in the long wavelength experiments varies from a few parts in 10^4 up to perhaps one part in 10^3. As further experience with the system is gained, a truly useful radiation source should result.

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Microwave Generation Using Sheet Relativistic Electron Beams

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***Abstract*—Potential advantages in the use of a sheet electron beam for generation of high-power microwave signals are discussed and preliminary experiments to establish their applicability are reported. An examination indicates that sheet beams probably have greatest utility in the frequency range 10–100 GHz.**

THE use of intense relativistic electron beams for the generation of high-power microwave signals has resulted in the development of gigawatt sources at frequencies of less than 10 GHz and megawatt sources at about 100 GHz [1]–[7]. These sources enhance by better than two orders of magnitude [8] the powers available from single source systems. Techniques for the generation of these high-power sources fall into two broad categories; axial bunching devices [1]–[3] such as slow wave systems, and transverse bunching systems such as the cyclotron [4]–[7] maser. The available power from these systems scales approximately as $1/f^{5/2}$ with the wave frequency [8]. At this meeting power levels [9] of about 1 MW at a wavelength of 0.5 mm have been reported and the generation mechanism identified as Raman scattering [10] from fluctuations in the electron beam. Since this generation technique depends on the scattering of a pump wave from an electron beam, the guide dimensions are determined by the beam pump requirements.

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